

The Hubble Law*

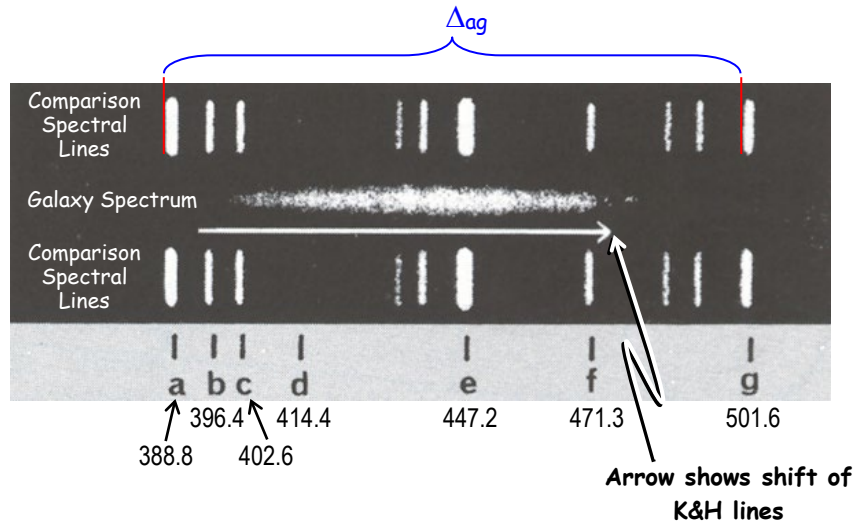
The prominent spectral lines of galaxies are redshifted due to the Doppler effect, produced by an object receding from the observer. This was described by Edwin Hubble as the *Hubble Law*,

$$v = H_0 d$$

where v is the recessional velocity of the galaxy (in kilometers per second), d is its distance and H_0 is the Hubble Constant. H_0 is a measure of how rapidly the Universe is expanding.

Scaling Factors

In the photos on p. 4 and the one to the right, the galaxy's spectrum is the blur in the middle with comparison spectra above and below showing lines of known wavelengths. The spectra of the galaxies each show two prominent dark lines, the K and H lines of ionized calcium at $\lambda_K = 393.4$ nm and $\lambda_H = 396.9$ nm. The arrow represents the amount by which the K and H lines have been redshifted using the wavelength between them, $\lambda_{KH} = 395.1$ nm.



- ★ Measure the distance (in mm) between two widely spaced lines (such as a and g).
- ★ From the diagram above, find the wavelengths for the comparison lines used, λ_{left} and λ_{right} .
- ★ Calculate the wavelength scale by finding the ratio of $\lambda_{right} - \lambda_{left}$ to distance D .⁵

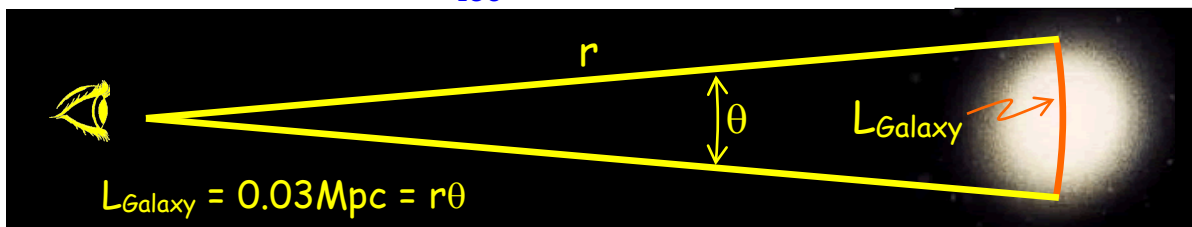
Distance: $D_{a \text{ to } g} = \underline{75.5}$ mm, Wavelengths: $\lambda_{left} = \underline{388.8}$ nm, $\lambda_{right} = \underline{501.6}$

$$\text{Wavelength Scale} = \frac{\lambda_{right} - \lambda_{left}}{D_{a \text{ to } g}} = \frac{112.8}{75.5} = 1.494 \frac{\text{nm}}{\text{mm}}$$

- ★ Measure the length (in mm) of the 150" line at the bottom of the page.
- ★ Calculate the size scale by finding the ratio of the angular size to measured size³

Length of 150" Line: $L_{150''} = \underline{24.6}$

$$\text{Size Scale} = \frac{150''}{L_{150''}} = \frac{150}{24.6} = 6.10 \frac{\text{arcsec}}{\text{mm}}$$



* After Evans, A., "Laboratory Exercises in Astronomy - Hubble's Law" *Sky & Telescope* April 1978

Recessional Velocities

For each spectrum on the diagram,²⁰

- ★ Measure the length of the arrow in **millimeters** showing the shift of the K & H lines.
- ★ Convert the measured shift to wavelength shift by using your scale:

$$\Delta\lambda_{K\&H} = (L_{\text{Arrow}})(\text{Wavelength Scale})$$

- ★ Find the recessional velocity of the galaxy by use of the equation to the right where c is the speed of light, $c = 3.0 \times 10^5 \text{ km/s}$.

$$v = \frac{\Delta\lambda_{\text{Galaxy}}}{\lambda_{KH}} c$$

Galaxy	Length of Arrow mm	Wavelength Shift $\Delta\lambda = (L_{\text{arrow}}) \times (\text{W.S.})$ nm	Recessional Velocity $(\Delta\lambda/\lambda_{KH}) \times c$ km/sec
Virgo	2	3.0	2,267
Ursa Major	14	20.9	15,871
Corona Borealis	21.5	32.1	24,374
Boötes	34.8	52.0	39,452
Hydra	53.8	80.4	60,992

Galaxy Distances

Assume all the galaxies are the same size so their apparent sizes are proportional to their distances³⁰

- ★ Measure the width of each galaxy in millimeters.
- ★ Convert the size to angular width by using your scale: $L_{\text{arcsec}} = L_{\text{mm}} (\text{Size Scale})$
- ★ Convert the arcseconds to radians using the fact that $206,265'' = 1 \text{ radian}$
- ★ Convert to distance assuming $L_{\text{galaxy}} = 0.03 \text{ Mpc}$ and

$$r_{\text{galaxy}} = \frac{0.03 \text{ Mpc}}{L_{\text{radians}}}$$

Galaxy	L_{max} mm	L_{min} mm	$\langle L_{\text{mm}} \rangle =$ $(L_{\text{min}} + L_{\text{max}})/2$ mm	$L_{\text{arcsec}} = \langle L_{\text{mm}} \rangle (\text{SS})$ arcsec	$L_{\text{radians}} = L_{\text{arcsec}}/206,265$ radians	Distance $= 0.03 \text{ Mpc} / L_{\text{radians}}$ Mpc
Virgo	24	18	21	131.1	6.36×10^{-4}	47.2
Ursa Major	6.5	5	5.75	21.3	1.03×10^{-4}	289.9
Corona Borealis	2.8	2.6	2.7	18.3	8.87×10^{-5}	338.3
Boötes	2	2	2	11.0	5.32×10^{-5}	563.8
Hydra	1.4	1	1.2	7.0	3.40×10^{-5}	882.5

Calculating the Hubble Constant:

Use Excel to plot data for your galaxies

- ★ Open T:\astronomy\Hubble and save it to your own One Drive
- ★ Type your values into the **white boxes**
- ★ Don't type on colored boxes!!
- ★ Calculations will be made as you type.

- ★ Your Hubble plot will appear at the bottom
- ★ The equation for your line will appear on the plot with numbers instead of ##.

$$y = ##x - ##$$

- ★ The coefficient of x is the slope which is your value of H_0 , write it here and type it into the bottom white box²⁰

$$H_0 = \frac{71.331 \text{ km/s}}{\text{Mpc}}$$

The current value of H_0 is about 72 km/s/Mpc. How does your value compare?

This is fairly close!

The Hubble Time

The age of the Universe is approximately the inverse of the Hubble constant; $1/H_0$. But we must convert the units. For your H_0 , the time will be calculated using 1 Mpc = 3.09×10^{19} km and 1 yr = 3.1536×10^7 sec. Write your value below and comment on it. How could you improve your measurements?

$$t_{\text{Hubble}} = \frac{1}{H_0 \left(\frac{\text{km}}{\text{Mpc-sec}} \right)} \left(\frac{3.09 \times 10^{19} \text{ km}}{1 \text{ Mpc}} \right) \left(\frac{1 \text{ yr}}{3.1536 \times 10^7 \text{ sec}} \right) = \frac{9.798 \times 10^{11}}{H_0} \text{ years}$$

Age of The Universe = 13.71 Billion years

How does this compare to the currently accepted 13.8 Billion year age of the universe?

This is pretty close! The measurements could be improved by figuring out a way to more consistently measure the sizes of the galaxies. The fuzziness makes it hard to be consistent.

Print out a copy of your spreadsheet page and attach it ... print to SLU_Public_Printing_color.

Calculation & Plotting of the Hubble Constant (S&T Exercise)

Find the Wavelength Scale
 λ_{rest} (a-g) 112.8 nm
 mm #DIV/0! nm/mm = W.S.

Find the Size Scale
 angle 150 arcsec
 mm #DIV/0! asec/mm = S.S.

$\lambda_{0, \text{KH}} = 395.1$ and $\lambda_{\text{observed}} = \lambda_{0, \text{KH}} + \Delta\lambda$
 $c = 3.00E+05$ km/s

	Arrow	$\Delta\lambda$	$\lambda_{\text{observed}}$	Speed
	L	$L*(\lambda/D)$	$L_0 + \text{Shift}$	$(\text{Shift}/L_0)*c$
	mm	nm	nm	km/s
Virgo		#DIV/0!	#DIV/0!	#DIV/0!
Ursa Major		#DIV/0!	#DIV/0!	#DIV/0!
Corona Borealis		#DIV/0!	#DIV/0!	#DIV/0!
Boötes		#DIV/0!	#DIV/0!	#DIV/0!
Hydra		#DIV/0!	#DIV/0!	#DIV/0!

These are your measurements from the front page.

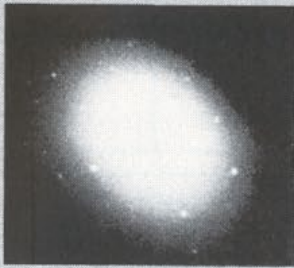
	Galaxy Size			Angle on sky		Distance
	Maj Axis	Min Axis	Average	arcsec	radians	
	mm	mm	mm			Mpc
Virgo			0	#DIV/0!	#DIV/0!	#DIV/0!
Ursa Major			0	#DIV/0!	#DIV/0!	#DIV/0!
Corona Borealis			0	#DIV/0!	#DIV/0!	#DIV/0!
Boötes			0	#DIV/0!	#DIV/0!	#DIV/0!
Hydra			0	#DIV/0!	#DIV/0!	#DIV/0!

Hubble Expansion

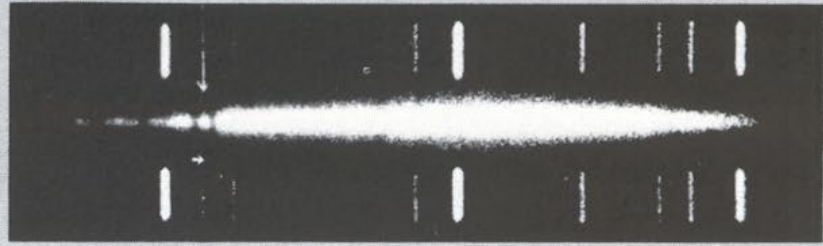
H_0 from plot = km/s/Mpc Type in from equation on plot

Hubble Time = years = billion yr
 Your age of the universe.

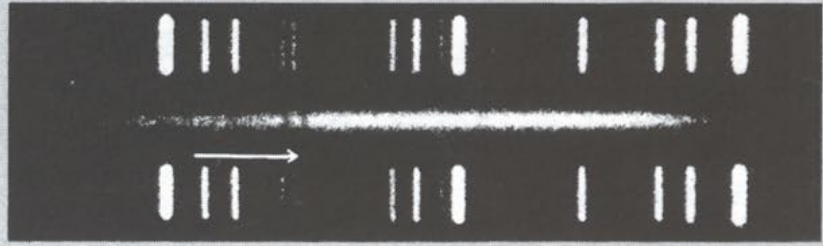
K and H lines



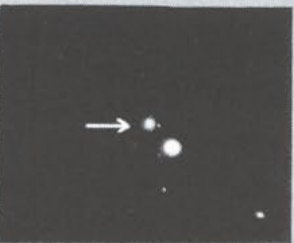
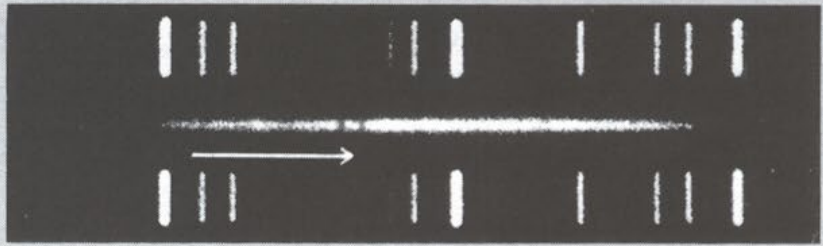
Virgo



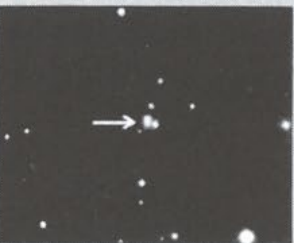
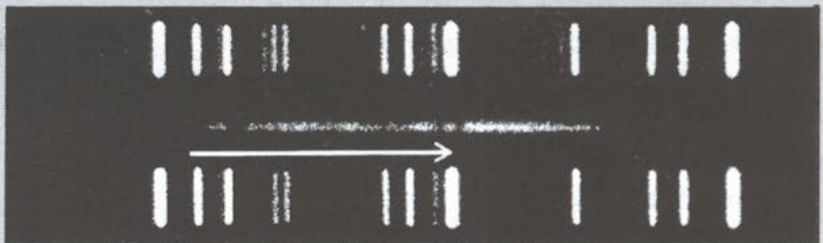
Ursa Major



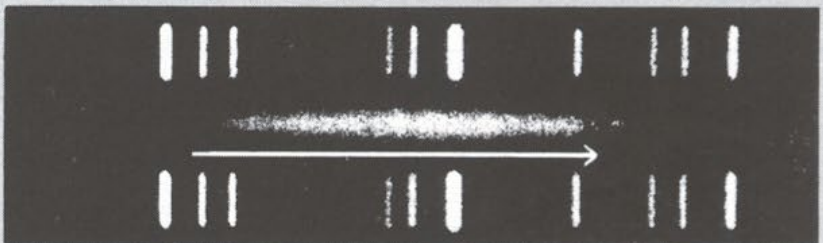
Corona Borealis



Bootes



Hydra



a b c d e f g

150"

Arrows indicate shift for calcium lines K and H.